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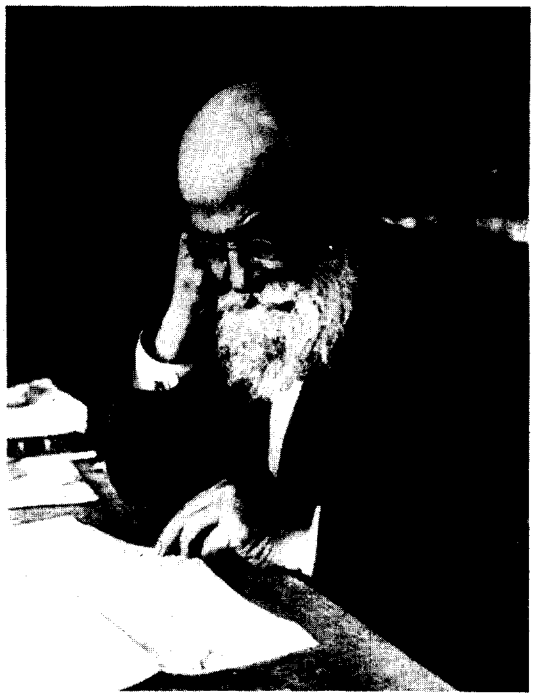

NIKOLAI YEGOROVICH ZHUKOVSKII† On the 150th anniversary of his birth (5(17) January 1847–17 March 1921)

G. Yu. Stepanov

In his bright and powerful personality he combined the sciences of both higher mathematics and engineering. He was the best combination of Science and Technology; he was practically a University. Undistracted by anything transient, recognizing the exigencies of life only when unavoidably necessary, he devoted all his extraordinary powers to scientific research. His entire nature was devoted heart and soul to that purpose. Herein lies the explanation for the enormously rich legacy that he has left us.

S. A. Chaplygin, 21 *March* 1921

The legacy of N. Ye. Zhukovskii (or Joukowski, according to the first French publications) is indeed enormous. It includes not only his published works and lecture courses, but also the publications of his school, his disciples and followers; it also includes the Central Aero- and Hydrodynamics Research Institute (CAHRI) and the Air-Force Academy (AFA), both of which he founded; and, later, the Central Research Institute of Aircraft Motor-Building (CIAMB), organized on the basis of the Propeller-Engine Department of CAHRI, and the Moscow Aviation Institute set up on the basis of the Aero-Mechanics Faculty of the Moscow Higher Technical School.

Zhukovskii's writings are represented by his *Complete Works* (1935–1939) in nine volumes and seven lecture publications. His life and activity have been treated in biographies written by his niece Ye. A. Dombrovskaya (1939), his former student and then close associate L. S. Leibenzon with the participation

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of N. M. Semenova (1947), in biographical books by A. A. Kosmodem'yanskii (1969, 1984) and M. S. Arlazorov (1959, 1964), and in more than 50 anniversary and thematic articles in various publications, including several articles by V. V. Golubev and articles in B. S. Stechkin, B. N. Yur'yev, I. I. Artobolevskii, S. A. Khristianovich, B. N. Delone, V. P. Vetchinkin and others. Zhukovskii's scientific results are reflected in the proceedings of various conferences and collections of articles on the History of Science, in particular: *The Development of Mechanics in the USSR* (1967), *Mechanics in the USSR after 50 Years* (vol. 1, 1968, and vol. 2, 1970), and *History of Mechanics from the End of the 18th Century to the Middle of the 20th Century* (1972).

In 1947, the then government and its scientific and engineering community celebrated the centenary of Zhukovskii's birth on a large scale. The Academy of Sciences, as well as the academic and scientific-technical councils of many organizations held centenary meetings, and leading journals devoted special articles to him. Our journal published a brief biography of Zhukovskii, a list of his 178 printed papers and a survey of them (*Prikl. Mat. Mekh.*, 1947, **11**, 1, 3–40). That same year saw the publication of centenary issues of Zhukovskii's *Selected Papers* (1948) in two volumes and of his *Collected Papers* (1948–1950) in seven volumes; the seventh volume included 12 articles reconstructed from manuscripts and six of Zhukovskii's reviews, as well as a list of 338 surviving titles of lectures delivered by him. Unfortunately, the index (tenth) volume of his *Complete Works*, prepared by A. P. Kotel'nikov in 1941–1943, has never been published; it was to have contained an index and abstracts of all Zhukovskii's works.

Zhukovskii's scientific research and teaching activities are inseparably linked with the Imperial Technical School (TS) and Moscow University (MU), in both of which he worked continuously until the end of his life.

Zhukovskii completed his studies at MU in 1868. That same year, having dreamt from childhood of taking up a practical engineering career, he began second-year studies in the Institute of Ways of Communications Engineers at St Petersburg. The difficulty of the studies, straitened circumstances and, finally, failure in a geodesy examination, forced him to give up his studies at that Institute. He returned to Moscow and, in the spring of 1871, took Master's examinations in mathematics and mechanics at MU, which enabled him to obtain a position as instructor at the TS. After defending his Master's thesis in 1879, Zhukovskii became head of the Department of Mechanics at the TS. In 1882 he defended his doctoral thesis at MU; within three years he had become Privat-Dozent in the Faculty of Mechanics at MU, and in 1886 he took over the Chair from F. A. Sludskii.

In the nineteenth century, mechanics had been developed mainly as an analytical science, along the path set by the schools of Lagrange and Cauchy, whose goal was to obtain and improve general theoretical results, with almost no bearing on the solution of particular problems and technological applications. The leader of these schools in Russia was the former Cauchy disciple M. V. Ostrogradskii. In university courses, mechanics was regarded as applied mathematics, and only P. L. Chebyshev in St Petersburg and Zhukovskii in Moscow were attracted in their scientific research and teaching activities to the solution of practical problems.

Following Poinsot in general mechanics and Helmholtz in hydrodynamics, Zhukovskii widely used and developed his own easily visualized geometrical notions and modelling as a fruitful method of research and study. He carried out his modelling not only using laboratory instruments, but also using analogue-physical and mental models; he used the same principles when attacking the mathematical formulation of the problems he studied. With consummate art, he simplified the scheme of a phenomenon to the point that it was possible to obtain simple and easily visualized results. He was rarely interested in general theoretical problems; the problems he solved were, as a rule, completely practical, taken from observations or experiments, and their solutions were reduced to numerical examples, experimental verification and technological applications.

Zhukovskii's publications encompass almost all the theoretical and applied mechanics of his time. His *Complete Works* include 189 separately titled works, printed together with editorial additions in 6400 pages. About half of these pertain to general (theoretical) and applied mechanics, a quarter to general and applied hydromechanics, and a quarter to aviation.

General and applied mechanics. Zukovskii's best known work in general mechanics is his doctoral thesis, "On the firmness of motion" (1882). Referring to the first attempt to establish a general theory of the "firmness" (stability) of motion, by Thomson and Tait in their *Natural Philosophy* (1867), and mentioning Routh's book of 1877, which was not known to him at the beginning of his research, Zhukovskii developed a linear theory of stability for the trajectories of conservative dynamical systems, now known as orbital stability. He sets up the equations of perturbed motion using the principle of least action in Jacobi's form, retaining in the equations only terms of the first order of smallness; a stability condition is then presented in a geometrical form, a "measure of the firmness of motion" is introduced and the theory is used to solve several problems, in particular, problems concerning the motion of a heavy Lagrange top and the motion of three-point masses in the Laplace cases, where the interaction is governed by an arbitrary power law. However, the honour of laying the foundations for the modern theory of the stability of motion in rigorous form goes to A. M. Lyapunov, who, in 1892 at MU, defended his remarkable doctoral thesis "A general theory of the stability of motion". One of his opponents in relation to this thesis was Zhukovskii. Later, in his lectures "The theory of machine operational control" (1906–1909), Zhukovskii gave a detailed account of the working principles and theory of all regulators of steam and hydraulic engines known at that time, calculated the effect of Coulomb friction, and studied the difference equations of control theory (for machines with steam cut-off). In these lectures Zhukovskii also mentions the fundamental work of I. A. Vishnegradskii (1876), but only in a polemical context, reflecting the debate that arose at the end of the century on the legitimacy of Vishnegradskii's linearization of friction forces.

Zhukovskii's work "A geometrical interpretation of the case treated by S. V. Kovalevskaya of the motion of a rigid body about a fixed point" (1896) may be reckoned among the most beautiful results of analytical mechanics. He presented there a geometrical treatment of the integrals defining the motion of a body, introduced appropriate orthogonal coordinate systems and described the mechanism that kinematically realizes the motion of the body in the Kovalevskaya case. In so doing, he completed the geometrical analysis, begun by Poinsot, of all possible cases of the motion of a body with a fixed point that admit of general integrals of motion in elliptic functions.

His contacts with F. A. Bredikhin, then professor at MU and director of the Moscow Observatory, led Zhukovskii to the solution of some astronomical problems (1881–1886) and, possibly, to his outstanding investigation of the motion of a rigid body having cavities filled with a homogeneous incompressible fluid (1885). In that research, generalizing partial results of Stokes, Helmholtz and Neumann, he presented an exhaustive study of the problem, with a definition of interior apparent masses and, in cases of multiply-connected cavities, interior angular momentum; he also considered examples of cavities of different shapes and studied the effect of the viscosity of the liquid filling the cavities.

Zhukovskii wrote a large number of papers on applied mechanics. Among his topics were the sliding of belts on pulleys, the flexible shaft of a turbine and shafts with vibrating bearings, the pressure distribution over the threads of a bolt and nut, the dynamics of trains and automobiles, etc.

Zhukovskii carried out interesting and original work on the conditions of equilibrium of a body when there are dry friction forces, being the first to investigate the special features of the loss of equilibrium due to friction forces obeying Coulomb's law.

Special mention should be made of Zhukovskii's important work on the theory of mechanisms and machines. In 1883 he completed Chebyshev's purely analytical researches on the structure of a plane four-bar chain with a point whose motion deviates minimally from a straight line, providing a lucid kinematic solution based on the requirement that the point must coincide with the centre of higher-order accelerations; in 1909 he proposed the "Zhukovskii lever", which enables the dynamic problem of determining the motion of any mechanism with one degree of freedom to be reduced to the problem of the equilibrium of its velocity scheme as a rigid system of rods. His research in this field was continued by his students A. V. Assur and N. I. Mertsalov, and then by I. I. Artobolevskii and others.

Zhukovskii's lecture courses on mechanics, which went through several editions, were fundamental for most teachers of theoretical mechanics and engineers in Russia up until the middle of the present century. As a teacher at the TS, he worked out what was to become a classical exposition of mechanics, in a simple, clear and at the same time rigorous form, with an abundance of illustrations, examples and exercises. He began to teach at MU at the age of 38, having had 15 years of experience as a professor at the TS and having formed definite views on research methods and teaching techniques in mechanics. Embarking on university teaching, Zhukovskii completely reconstructed it on the basis of his own experience. To second-year students he delivered an introduction to mechanics (kinematics and geometrical statics) and the dynamics of a point mass, while third-year students took courses on analytical statics, dynamics of systems and the theory of attraction. The first semester of the fourth year was taken up by lectures on hydrostatics and the elements of hydrodynamics, and the second by specialized courses on analytical mechanics and the dynamics of a non-variable body. When Zhukovskii became head of the Mechanics Faculty, he made it his laboratory, gradually converting it to aerodynamics.

Zhukovskii's courses on mechanics are formulated in coordinate notation, though he always draws the vectors of forces, couples, velocities and accelerations and uses vector addition. Zhukovskii bases the dynamics of a point on three laws: Galileo's law of inertia, the law that the forces are independent of motion and act independently, and the equality of action and reaction. Following the authors of French textbooks, he calls the forces of inertia "fictitious", which nevertheless does not prevent him assuming that in all problems of dynamics they "in no way differ from real forces" and, accordingly, that by d'Alembert's theorem (in a kinetic-static formulation) "the forces of motion together with the forces of inertia are in equilibrium with the resistance of the constraints in the system". Even today, much of the material in Zhukovskii's courses deserves the attention of teachers of theoretical mechanics and textbook authors: the motion of a projectile with a square law of resistance, the isochronous (cycloidal) pendulum, regulators of the operation of machines, catenaries and threads of constant strength, as well as Zhukovskii's exposition of the Lagrange and Hamilton principles, the geometrical interpretation of the free motion of a body and motion in the Kovalevskaya case, and the theory of potential of attraction of an ellipsoid and an ellipsoidal shell.

General and applied hydrodynamics. The development of hydrodynamics in the nineteenth century, unlike the mechanics of a deformable rigid body, had almost nothing to do with technological applications. Leaving answered the main question, as to the origin of the forces of interaction of a fluid and a body, the Euler–d'Alembert paradox that there are no such forces in the steady motion of bodies in an ideal fluid cast doubt on the practical sense of the model, and theoretical work, as well as courses of study, took the form of mathematical exercises. The equations of motion of a viscous fluid could not be integrated except in very special cases, and Helmholtz's jet theory, though resolving the Euler–d'Alembert paradox to some extent, turned out to be groundless for calculating real forces and created a new paradox: the wake of a body moving in a stationary fluid would have infinite kinetic energy.

A major scientific result was his well-known theorem on the relationship between lift and velocity circulation. This theorem, which resolved the Euler–d'Alembert paradox once and for all, marked a veritable revolution in the development of hydrodynamics and placed Zhukovskii's name in the same ranks as the founders of that discipline.

Zhukovskii's first published work was in hydrodynamics—his Master's thesis, "Kinematics of a fluid body" (1876). Five years of intensive work acquainted him with the contemporary state of the art and in particular with the works of Thomson and Tait, Helmholtz, Bertrand and Kirchhoff. In his thesis, he presented "a clear outline of the theory of the velocities of a continuously varying body", in the general case of the three-dimensional vortex motion of a compressible fluid, using geometrical concepts and natural coordinates. He described the motion in an infinitesimal particle, considered the streamlines, gave a classification of singular points, defined the circulation of the velocity following Thomson, and reduced the equations of the absence of vortices and continuity to natural coordinates using the curvature of the streamlines and the orthogonals to them.

He subsequently published three further works on hydrodynamics (1876, 1884, 1885), two of which, also concerned with mechanics, have already been mentioned, and, finally, his remarkable *Lectures on Hydrodynamics*, which he wrote and read at MU in 1886. These lectures constitute a comprehensive (for that time) exposition of hydrodynamics. After an account of kinematics, he derives the equations of fluid motion, considers the electrohydrodynamic analogy, introduces complex variables, uses elliptic coordinates and presents examples of motion with constant vorticity.

In 1890, he published his fundamental study, "A modification of Kirchhoff's method for determining the motion of a liquid in two dimensions at constant velocity, given on an unknown streamline". Unlike Kirchhoff (1869), Zhukovskii does not uses the complex flow velocity but its logarithm, the "Zhukovskii function", and considers it and the complex potential as analytic functions of an auxiliary variable which varies in the upper halfplane. Using this method, he solved 18 problems, ten of them for the first time. Of particular importance is his construction of jet flows with finite lengths of the free boundaries. One of these flows was used in 1955, simultaneously and independently, by Roshko and Eppler, as a unified computational scheme for separated and cavitation flows (the "Zhukovskii–Roshko–Eppler scheme"). In addition, he presented the first solution of a problem in the theory of hydrodynamic cascades—the problem of jet flow around a cascade of plates when the free stream is perpendicular to the cascade front (a complete solution of this problem was published by Chaplygin and Minakov in 1930).

In his next paper, "Determination of the motion of a liquid under any condition given on a streamline" (1891), he gave the first formulation and rigorous solution of two now classical problems in the jet flows of heavy and capillary liquids.

The outstanding importance of these two papers lies not only in their presentation of a new method in the jet theory of an ideal fluid and the solution of many new problems, but also in the formulation of several new ideas which set the stage for the work of the Moscow school on the theory of functions of a complex variable in hydro- and aerodynamics (the most prominent representative of that school would be S. A. Chaplygin, followed by V. V. Golubev, M. V. Keldysh, L. I. Sedov and M. A. Lavrent'yev).

In a paper entitled "On bound vortices", delivered at the Moscow Mathematical Society in 1905 and published in 1906, Zhukovskii, by applying the momentum theorem to a contour at an infinite distance from a cylindrical body, established his remarkable theorem according to which the lift acting on a body in an infinite steady plane-parallel potential flow is perpendicular to the flow direction and equal in magnitude to the product of the fluid density, the free-stream velocity and the circulation of the velocity about the body (Zhukovskii had in mind here the circulation of bound vortices rather than free vortices, which by assumption are not present in the flow).

The "Zhukovskii force" had been calculated in particular cases by Lord Rayleigh (1878) for flow past a circular cylinder and by Kutta (1902) for an arc of a circle in a smooth flow, but only Zhukovskii, for the first time, published an expression for the lift in general form, valid for any body or system of bodies, and his priority in this respect is beyond doubt.

In a subsequent paper "On the descent in air of light elongated bodies rotating about their long axes", which appeared in the same year, Zhukovskii repeated the derivation of his formula, taking into account not only bound vortices but also bound sources, whose thrust is expressed by a similar formula (with the source strength instead of the circulation); both these formulae should thus be referred to as "Zhukovskii's theorem".

Asymptotically, on the scale of an infinitely distant contour, any body is strongly equivalent, in respect of its principal vector and the principal moment of the acting forces, to a vortex–source combination situated at a certain point. It is in fact "bound" in Zhukovskii's sense to a vortex–source combination (or to a vortex alone if there is no source).

As is now well known, Zhukovskii's theorem holds without modification for the barotropic motion of a gas (this was demonstrated for low subsonic velocities by Keldysh and Frankl' in 1934, and for arbitrary velocities by Sedov in 1948).

The subject range of Zhukovskii's publication in applied hydrodynamics, as in mechanics, is very broad.

In 1899, at the suggestion of a colleague at the TS, N. P. Zimin, Zhukovskii took part in experimental investigations of transient pressures in the pipes of an Alekseyev water conduit, culminating in his remarkable paper "On hydraulic shock in water pipes". In that paper Zhukovskii developed a theory of the phenomenon in a one-dimensional acoustic formulation and derived a formula for the pressure jump, which equals the product of the liquid density, the velocity lost in the shock and the velocity of sound, which depends on the liquid compressibility and the elastic properties of the pipes. He fully explained all the effects that were observed, recommended safe rates of valve-closing and a technique for determining the positions of liquid leaks or gas build-up in pipes, based on indicator diagrams. This now world-famous work, apart from its triumphant technical significance, furnished proof, in addition to Mach's shadow photography, of the fact that strong discontinuities (jumps) actually exist in compressible fluids.

Influenced by the work of N. P. Petrov, Zhukovskii embarked on a hydrodynamic investigation of fluid-lubricant bearings. In two papers, published in 1886 and 1887, he improved Petrov's theory and then, in 1904, in collaboration with Chaplygin, presented the first rigorous solution of Stokes' equations in a lubricating layer, valid for any clearance and eccentricity of the bearing. (Here Chaplygin's idea of expressing the general solution of a biharmonic equation in terms of two arbitrary analytic functions—which he proposed independently of Goursat around 1900—was essential.)

Zhukovskii's papers on ground water-flow theory, published in 1899, 1890, 1906 and 1923, may also be categorized as dealing with applied problems of hydrodynamics. They contain the derivation of the basic equations of ground water-flow theory and the first solutions of several particular problems. The last paper introduced a new function, the "Zhukovskii function", which reduced the problem of plane-parallel flows of a heavy liquid in porous media when there are free surface (at constant pressure) to problems of jet theory for an ideal, weightless fluid. These papers laid the foundation for the Russian school of filtration theory (N. N. Pavlovskii, L. S. Leibenzon, S. N. Numerov, P. Ya. Polubarinova-Kochina, V. N. Shchelkachev, and others).

The beginning of the twentieth century saw the emergence of boundary-layer theory, whose equations were developed in 1904 by Prandtl. Neither Zhukovskii nor Chaplygin were interested in that theory, and only in the 1930s did members of their school publish papers that were later to make a major contribution to boundary-layer theory (L. S. Leibenzon, V. V. Golubev, L. G. Loitsyanskii, K. K. Fedyayevskii, and others).

Aerodynamics. It is hard to say when Zhukovskii first took an interest in problems of flight. Man's dreams of flying are as old as Greek mythology. By the end of the last century, engineers in different countries were making attempts to design heavier-than-air flying machines. In 1903 the Wright brothers in the USA and in 1906 Santos-Dumont in Europe made the first successful controlled flights in heavier-than-air machines. This tremendous technological achievement outstripped the theory: at that time, mechanics could not properly calculate the lift of a wing or the thrust of a propeller. Ignorance of the pressure distribution over the wing surface made it impossible to evaluate the strength and stability of aircraft in flight; the first constructions were worked out by trial and error, sometimes with catastrophic results.

Zhukovskii's first publication in aerodynamics, "The theory of flight", belongs to the year 1890. He was concerned there with "the ancient problem of fulcrum" and pointed out, as sufficient, two ways of producing thrust: the formation of vortical surfaces of discontinuity of the velocity (Helmholtz–Kirchhoff jets) and the use of the friction of moving surfaces. Zhukovskii had also considered another way of producing thrust—by using hydro-jet propulsion—in papers of 1882 and 1886 and later in 1908. According to a formula of Bernoulli, the thrust produced by such jet engines is the mass flow multiplied by the difference between the velocity of the craft and the relative rate of water ejection. Zhukovskii undoubtedly had the idea of an "air-jet engine", whose thrust would be calculated by a similar formula. To quote Leibenzon, in 1903–1905 Zhukovskii designed and constructed a model of an "air thermal jet engine" fueled by alcohol. Two such engines were mounted at the ends of the propeller blades. The experiments failed, however, and Zhukovskii never returned again to these two ideas, in both of which he was ahead of his time—a single-pass air-jet engine ("ramjet") and a jet-driven propeller; he used Bernoulli's formula again only to calculate propeller thrust (1907).

His growing interest in the problems of flight is reflected in a large number of articles and lectures in which he considered the mechanics of the soaring of birds and discovered the possibility of a "dead loop" (1891), demonstrated Mach's photographs of a flying bullet (1891), and spoke on the flying machines appearing at that time and of Lilienthal's apparatus, experiments and flights, of multi-screw helicopters, etc. In 1898, at the tenth conference of Russian Naturalists and Physicians, he made a pronouncement that was to become popular: "Man will fly by relying not on the strength of his muscles but on the power of his wits".

Zhukovskii's theorem, discovered in 1906, offered the first correct explanation of the production of lift, but it was not immediately applied to wing theory: it was not clear how to define one of the quantities in his formula, the circulation of the velocity.

The year 1910 saw the publication of Chaplygin's papers and, a little later, a paper by Zhukovskii in which, albeit by different methods, they constructed the flows around theoretical airfoils with one sharp trailing edge and a finite velocity on it (that very condition defined the magnitude of the circulation). They assumed, as a quite natural condition, that the velocity at the sharp angles of bodies (airfoils) projecting into the flow had to be finite and ascribed no special significance—or designation—to that condition. In most publications this condition is known as Zhukovskii's hypothesis or the "Zhukovskii–Chaplygin condition", terms adopted by Zhukovskii's contemporaries; many authors from CAHRI and AFA, however, following Golubev, insist, without sufficient reason, on the terms "Chaplygin's hypothesis" or the "Chaplygin–Zhukovskii postulate" and associate it with the effect of viscosity.

In actual fact, this condition is by no means explained by the effect of viscosity—which does not exist in an ideal fluid—but only by a physically obvious restriction: a fluid is not capable of sustaining a high tensile force. The first explicit observation as to the need for this restriction in flow past sharp edges was made by Helmholtz, in a paper published in 1868 (a Russian translation, edited and annotated by Chaplygin, was published by "Pallas" in 1902). It was in that connection that Helmholtz suggested his jet scheme of the flow of an ideal fluid with the formation of vortical surfaces of discontinuity, a scheme that guaranteed finite velocities at sharp edges. Zhukovskii recalled and developed the idea in a paper entitled "On the question of cutting the vortex filaments" (1894), where he solved the problem of the motion of a free vortex in flow past a sharp edge. Besides this paper, he used the condition and drew appropriate diagrams several times before 1910 (beginning with the already mentioned paper "A modification of Kirchhoff's method. . . "); true, he did not immediately use the condition to construct the flow around airfoils. Accordingly, it would be historically accurate to refer to the condition that the velocity at the trailing edge of a wing be finite as the "Zhukovskii–Chaplygin condition", as is indeed done by L. S. Leibenzon, L. G. Loitsyanskii, N. Ya. Fabrikant, and others.

In 1910–1912 Zhukovskii continued to present papers and lectures on the most diverse questions of aerodynamics and aviation. Among them were descriptions of the aerodynamics laboratories at MU and the TS with diagrams of the equipment, photographs and experimental results. An enthusiastic Zhukovskii later prepared and delivered at the TS his famous course, "Theoretical foundations of aeronautics", which was immediately published by his listeners, and then also in Paris (1916 and 1931). In that course, as Zhukovskii stated in the Preface, he strove "to link the rich experimental material accumulated in aerodynamics laboratories with theoretical research on the relevant problems, using the equations of hydrodynamics".

An outstanding event in the development of airfoil theory was Zhukovskii's publication of a cycle of four papers on related subjects in 1910–1912. The main point in these papers was the proposal of an elegant geometrical device to construct theoretical airfoils by "rounding out" the "basis", first as an arc of a circle, then as a circular lune (Antoinette or Rateau airfoil). He obtained smooth drop-shaped airfoils with one infinitely thin trailing edge, using a long familiar conformal mapping which is known in this context as the "Zhukovskii transformation". However, the airfoils thus obtained had already been constructed in the same year by Chaplygin by parabola inversion, as Zhukovskii himself noted, it would be more correct to call them "Chaplygin–Zhukovskii airfoils". To obtain circular lunes and round them off, he used bipolar coordinates and a power-law transformation. Somewhat later, similar airfoils with wedge trailing edges were constructed by Karman and Treffitz (1918). Such airfoils should clearly be called "Zhukovskii airfoils".

Turning to an evaluation of resistance forces, Zhukovskii states that the main cause of such forces are trailing vortices. He constructs a model of the phenomenon in the form of a point vortex before the sharp leading edge of a thin airfoil in such a way that the edge is a critical point. Such a vortex is subject to a Zhukovskii force. As it moves, the Zhukovskii–Chaplygin condition is violated, causing the appearance of another vortex, and so on. As is evident, Zhukovskii's model conceptually anticipated modern calculations of separated flow around thin airfoils by the discrete vortex method. Zhukovskii, not confining himself to theoretical analysis, carried out experimental measurements in the wind tunnels of the TS and MU, with satisfactory confirmation of the calculation at low angles of attack.

Nowadays, theoretical airfoils, having no special hydrodynamic features, are not used as wings. Nevertheless, they are of permanent scientific and pedagogical value and can be used to test techniques for solving direct and inverse problems of hydro- and aerodynamics.

In 1912 Zhukovskii began to publish his celebrated vortex theory of screw propellers in four papers (1912–1918). Issues relating to the theory of propellers and turbo-machines had been attracting his attention for some time. His screw propeller theory extends his idea of these bound vortices to three-dimensional flow around the blades, in the form of free vortex "filaments" which come off into the flow from the ends and edges of the blades and from the hub of the propeller. Zhukovskii mentions a similar but simpler pattern of flow past a wing of finite span, referring to Lanchester's *Aerodynamics* (1907) and a survey paper by Finsterwalder (1910). He illustrates the appropriate Π-shaped vortex pattern, also referring to papers by Chaplygin (1910) and Prandtl (1912).

Zhukovskii determines conditions for vortex filaments to be stationary on the surface of a circular cylinder, calculates the velocities in the mean axisymmetric flow, replacing the propeller by a vortex disk, considers the selection of the propeller blades and determines the thrust and power of the propeller. He then studies the relationship between the axial and peripheral components of the velocities for blades designed using different laws of twist, in particular, he points out as most favorable the law of constant circulation along the blades (the "NEZh propeller"), extends his formula for wing lift to the airfoil in a cascade in an annular channel, referring to work by his former student and coworker V. P. Vetchinkin, he calculates the contraction of the jet behind the propeller and the shedding of vortices from the blade edges; he presents a new and very simple solution of the problem of continuous flow past a cascade of plates; finally, he presents the results of tests of propellers and fan ventilators at the TS laboratory. All four papers were translated and published in Paris (1929).

In these papers, apart from material with a direct bearing on screw propeller theory, one also finds many ideas of modern turbomachine theory; together with Chaplygin, Zhukovskii was undoubtedly a pioneer in solving the first problems of the theory of cascades.

In the last years of his life, acting as a consultant to the Commission of Special Artillery Experiments, he turned to problems of gas dynamics. He made a detailed investigation of the unsteady motion of a heavy liquid in an open channel and of gas in a pipe. One might say that these papers, together with the theory of hydraulic shock, laid the foundations for one-dimensional gas dynamics and for gas-hydraulic analogy. In F. I. Frankl's opinion, Zhukovskii possessed sufficient tools to derive the formulae of the linear theory of bodies in a supersonic flow and to calculate the wave resistance of projectiles and the lift of a wing in supersonic flow, but he was not to complete and publish these calculations.

Zhukovskii's work is now a part of history. His scientific legacy, however, and the remarkable Zhukovskii–Chaplygin school are a permanent memorial to him. Regardless of any temporary obstacles, we will always remember Zhukovskii's radiant appearance, masterfully captured by V. V. Golubev in the biographical sketch:

> A powerful figure, exceptional calm, a thoughtful, concentrated look, extraordinary simplicity, a natural air in his dealing with those around him, striking modesty and at the same time self-confidence and self-respect created a radiant image of a scholar and sage. Nikolai Yegorovich Zhukovskii observed the world around him with philosophical calm; with lively interest he thoroughly scrutinized all beliefs, without falling into useless arguments, striving to find something valuable, worthy of attention, in any opinion.

The extraordinary range of his scientific talents, the happy combination of geometrical intuition, a feeling of live reality and analytical art were combined with a broad view of people and of life. A stranger to any one-sidedness, he knew how to unite people of different views, diverse temperaments and varied talents and capacities in a common cause, each finding in his work a place to apply his particular powers and gifts. ...

Faith in the disciples surrounding him, a conviction of the profound value of scientific knowledge, a limitless and disinterested devotion to science—all these traits were combined in a man, a scholar and a teacher—a feature that will always live in his disciples' memory.

V. V. Golubev, 1935

Translated by D.L